

Table 22.2

Coefficients for the Ultraspherical Polynomials $C_n^{(\alpha)}(x)$ and for x^n in terms of $C_m^{(\alpha)}(x)$

$$C_n^{(\alpha)}(x) = a_n^{-1} \sum_{m=0}^n c_m x^m \text{ and } x^n = b_n^{-1} \sum_{m=0}^n d_m C_m^{(\alpha)}(x) \quad (\alpha \neq 0)$$

		x^0	x^1	x^2	x^3	x^4	x^5	x^6	
	b_n	1	2α	$2(\alpha)_2$	$4(\alpha)_3$	$4(\alpha)_4$	$8(\alpha)_5$	$8(\alpha)_6$	
$C_0^{(\alpha)}$	a_n 1	1	1	α		$3\alpha(\alpha+3)$		$15\alpha(\alpha+4)(\alpha+5)$	$C_0^{(\alpha)}$
$C_1^{(\alpha)}$	1		2α	1	$3(\alpha+1)$		$15(\alpha+1)(\alpha+4)$		$C_1^{(\alpha)}$
$C_2^{(\alpha)}$	1	$-\alpha$		$2(\alpha)_2$	1	$6(\alpha+2)$		$45(\alpha+2)(\alpha+5)$	$C_2^{(\alpha)}$
$C_3^{(\alpha)}$	3		$-6(\alpha)_2$		$4(\alpha)_3$	3	$30(\alpha+3)$		$C_3^{(\alpha)}$
$C_4^{(\alpha)}$	6	$3(\alpha)_2$		$-12(\alpha)_3$		$4(\alpha)_4$	6	$90(\alpha+4)$	$C_4^{(\alpha)}$
$C_5^{(\alpha)}$	15		$15(\alpha)_2$		$-20(\alpha)_4$		$4(\alpha)_5$	30	$C_5^{(\alpha)}$
$C_6^{(\alpha)}$	90	$-15(\alpha)_2$		$90(\alpha)_4$		$-60(\alpha)_5$		$8(\alpha)_6$	$C_6^{(\alpha)}$
		x^0	x^1	x^2	x^3	x^4	x^5	x^6	

$$(\alpha)_n = \alpha(\alpha+1)(\alpha+2) \dots (\alpha+n-1)$$

$$C_3^{(2)}(x) = \frac{1}{3} [4(2)_2 x^2 - 6(2)_2 x] \quad x^3 = \frac{1}{4(2)_3} [3(3) C_1^{(2)}(x) + 3C_3^{(2)}(x)]$$

$$C_3^{(3)}(x) = \frac{1}{3} [96x^3 - 36x] \quad x^3 = \frac{1}{96} [9C_1^{(3)}(x) + 3C_3^{(3)}(x)]$$